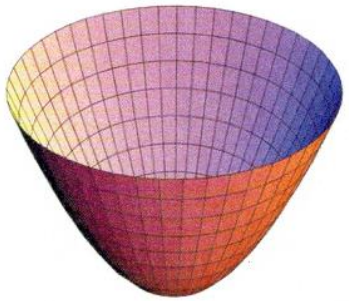
Module 1: Section 1D: A Closer Look at the Standards for Mathematical Content: High School Geometry Sample Tasks

Task 1:

Task

Charles and Olivia are trying to estimate the volume of water that could be held by the figure shown below, which is 10 feet high and has a circular top of radius 20 feet. Charles proposes they approximate the volume by using a cylinder of radius 20 feet and height 10 feet. Olivia proposes that they instead use a circular cone connecting the top of the tank to the vertex at the bottom.

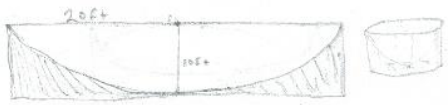


What answers would the two methods predict? Which is likely to be most accurate? What is your best estimate for the volume of the tank?

Charles' method

$V(\text{cylinder}) = \pi r^2 h = \pi (20)^2 10 = 4000\pi \text{ ft}^3$

$V(\text{cylinder}) \approx 12{,}566 \text{ ft}^3$

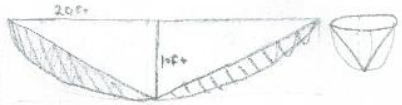


$V(\text{cylinder}) > V(\text{tank})$

Olivia's method

$V(\text{cone}) = \frac{1}{3}\pi r^2 h = \frac{4000}{3}\pi \text{ ft}^3$

$V(\text{cone}) \approx 4{,}189 \text{ ft}^3$



$V(\text{cone}) < V(\text{tank})$

Olivia's method would be more accurate. It will be less volume than the actual tank, but much closer than Charles' method will predict. My best estimate is the volume of the tank is about 8250 ft^3. Based on my sketches, I think the tank is about $\frac{1}{2}$ V(cone) more than Olivia's method and about $\frac{1}{2}$ V(cylinder) less than Charles's method.

Please note that inclusion of these sample tasks does not represent that this task is endorsed by or rejected by the Kentucky Department of Education. Inclusion of these tasks was for the sole purpose of allowing participants the opportunity to investigate the content standards within the *Kentucky Academic Standards for Mathematics* more closely. All tasks were selected from https://tntp.org/student-work-library.

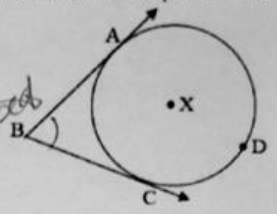
Task 2:

Proving the Circumscribed Angle Theorum

A circumscribed angle is an angle formed by two rays from a common endpoint that are tangent to a circle.

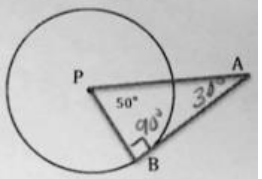
central angle m∠AXD

circumscribed angle



Circumscribed Angle Theorum
A circumscribed angle and a central angle are supplementary

1. In the figure, $\overline{AB}$ is tangent to circle P at point B. What is the measure of angle PAB?



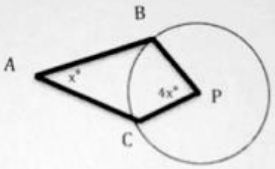
30°

2. $\overline{AB}$ is tangent to circle P at B and $\overline{AC}$ is tangent to circle P at C. Find the measure of angle A.

$x + 4x = 180$

$5x = 180$

$x = 36°$



3. $\overline{AB}$ is tangent to circle C at B.
$\overline{AD}$ is tangent to circle C at D.
The measure of arc ED is 134 degrees.

a. Find the measure of angle E.

$m\angle BCD = 180 - 50 = 130$

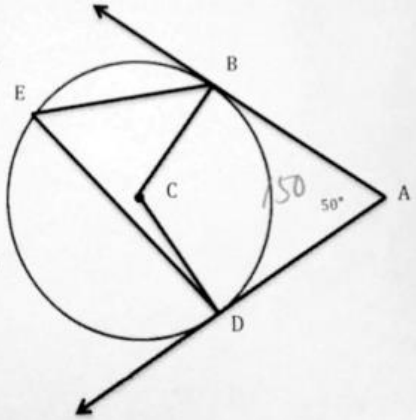
$m\angle E = \frac{1}{2}(130) = 65°$

b. Find the measure of arc EB.

$m\overset{\frown}{EB} = 362 - 264 = 78°$

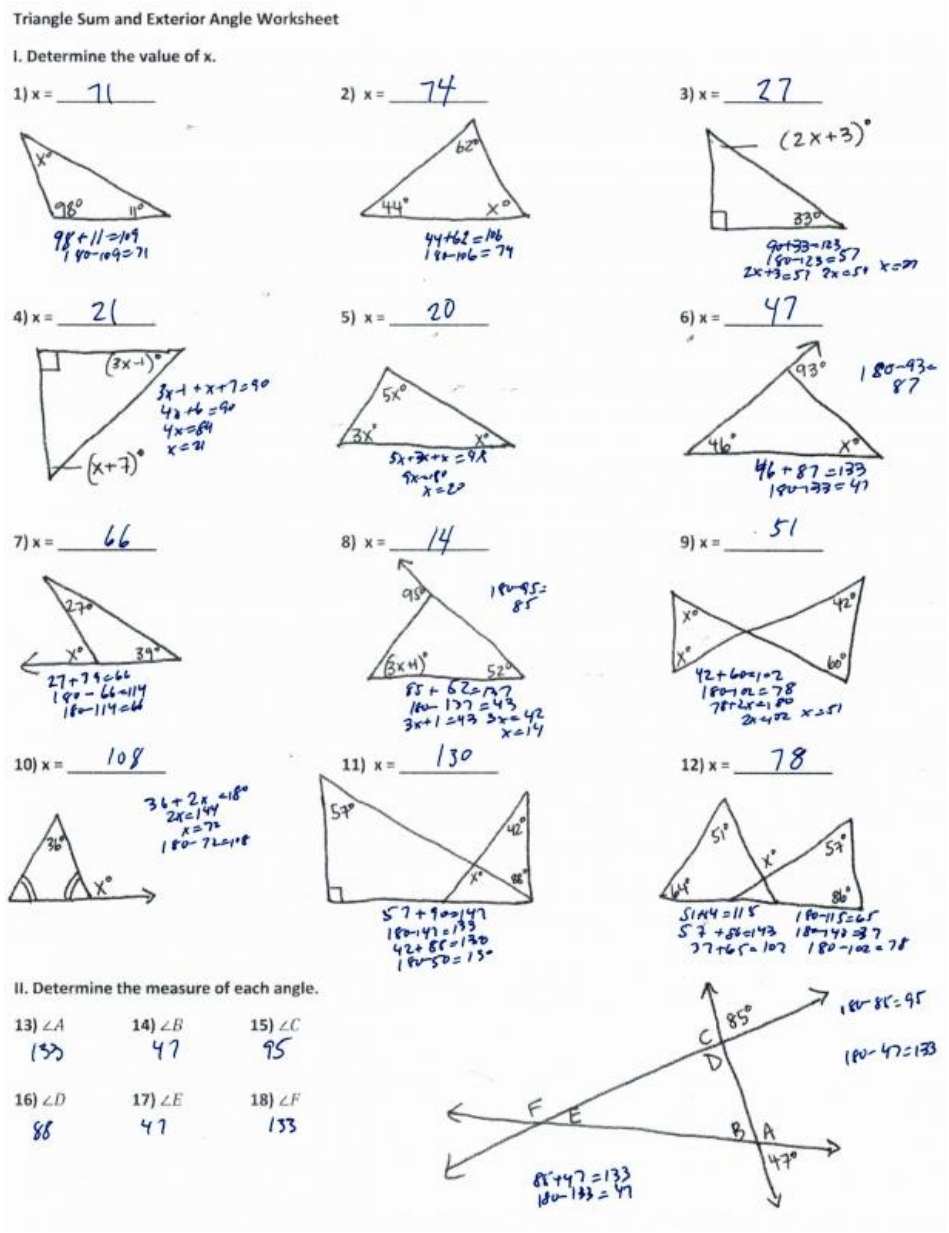
134 + 150

284



Please note that inclusion of these sample tasks does not represent that this task is endorsed by or rejected by the Kentucky Department of Education. Inclusion of these tasks was for the sole purpose of allowing participants the opportunity to investigate the content standards within the *Kentucky Academic Standards for Mathematics* more closely. All tasks were selected from https://tntp.org/student-work-library.

Task 3:



Triangle Sum and Exterior Angle Worksheet

I. Determine the value of x.

1) x = 71

2) x = 74

3) x = 27

4) x = 21

5) x = 20

6) x = 47

7) x = 66

8) x = 14

9) x = 51

10) x = 108

11) x = 130

12) x = 78

II. Determine the measure of each angle.

13) ∠A 133

14) ∠B 47

15) ∠C 95

16) ∠D 88

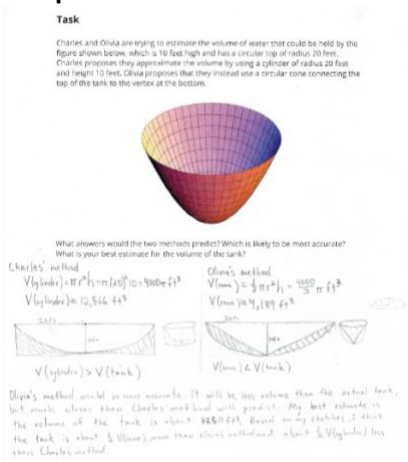
17) ∠E 47

18) ∠F 133

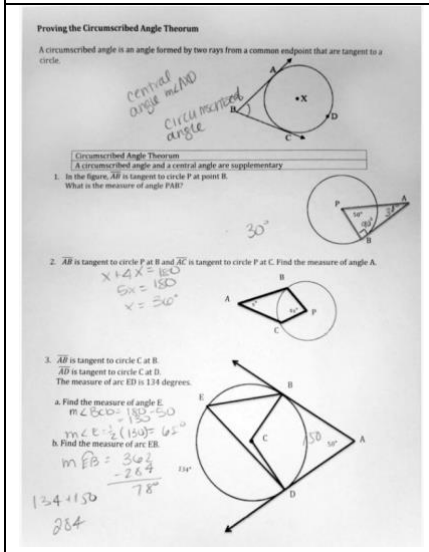
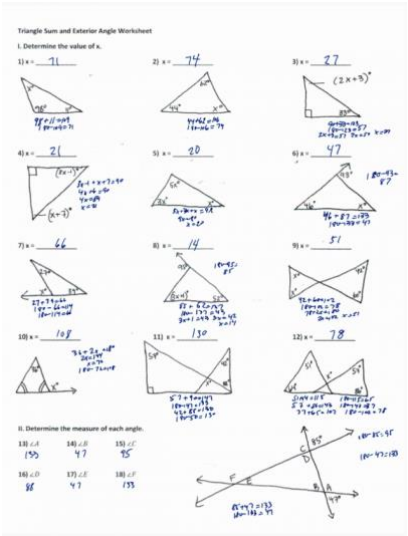
Please note that inclusion of these sample tasks does not represent that this task is endorsed by or rejected by the Kentucky Department of Education. Inclusion of these tasks was for the sole purpose of allowing participants the opportunity to investigate the content standards within the *Kentucky Academic Standards for Mathematics* more closely. All tasks were selected from https://tntp.org/student-work-library.

Module 1: Section 1D: A Closer Look at the Standards for Mathematical Content: High School Geometry Sample Tasks

Participant Guide

Student Work Sample	Standard of Mathematical Content Focus	Degree of Alignment	Standards of Mathematical Practice (SMP) Focus
Sample Task 1:  Task Charles and Olivia are trying to estimate the volume of water that could be held by the figure shown below, which is 10 feet high and has a circular top of radius 20 feet. Charles proposes they approximate the volume by using a cylinder of radius 20 feet and height 10 feet. Olivia proposes that they instead use a circular cone connecting the top of the tank to the vertex at the bottom. What answers would the two methods predict? Which is likely to be most accurate? What is your best estimate for the volume of the tank?	Can you identify the targeted content standard(s) for this task?	• None/Weak • Partial • Strong	Can you identify the targeted practice standard(s) for this task?
Sample Task 2:	Can you identify the targeted content standard(s) for this task?	• None/Weak • Partial • Strong	Can you identify the targeted practice standard(s) for this task?

Please note that inclusion of these sample tasks does not represent that this task is endorsed by or rejected by the Kentucky Department of Education. Inclusion of these tasks was for the sole purpose of allowing participants the opportunity to investigate the content standards within the *Kentucky Academic Standards for Mathematics* more closely. All tasks were selected from https://tntp.org/student-work-library.

Student Work Sample	Standard of Mathematical Content Focus	Degree of Alignment	Standards of Mathematical Practice (SMP) Focus
			
Sample Task 3: 	Can you identify the targeted content standard(s) for this task?	• None/Weak • Partial • Strong	Can you identify the targeted practice standard(s) for this task?

Please note that inclusion of these sample tasks does not represent that this task is endorsed by or rejected by the Kentucky Department of Education. Inclusion of these tasks was for the sole purpose of allowing participants the opportunity to investigate the content standards within the *Kentucky Academic Standards for Mathematics* more closely. All tasks were selected from https://tntp.org/student-work-library.

Module 1: Section 1D: A Closer Look at the Standards for Mathematical Content: Geometry Sample Tasks

Facilitator's Guide

Throughout facilitation of this activity it will be important to remind participants:

- Use the grade-level overview to determine the relevant cluster(s) to look at more closely
- Questions regarding Standards for Mathematical Practices will only be indicated where specific practices were identified within the source of the task alignment. Additionally, emphasize to participants the statement at the end of each cluster within the *KAS for Mathematics*, "The identified mathematical practices, coherence connections, and clarifications are possible suggestions; however, they are not the only pathways."

Sample Task 1:

This assignment is strongly aligned to the standards.

Overview

Students apply the volume formulas for a cylinder and a cone and reason about the model that provides the best estimate for a given geometric figure. This assignment is strong because it is well-aligned to a high school geometry standard. The task provides an opportunity to apply geometric volume formulas to solve a real-world problem and, as required by the standard, it involves students in the modeling process.

Related Standards

We looked at how well the assignment aligned to the following standard:
KY.HS.G.27*: Use volume formulas to solve problems for cylinders, pyramids, cones, spheres, prisms.

*The asterisk is included in the KAS for Mathematics and indicates that this is a modeling standard. Modeling is best interpreted not as a collection of isolated topics, but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice.

Why is this assignment Strongly aligned?

In eighth grade, students learn the volume formulas for cylinders and cones (standard KY.8.G.9). In high school, they can explain how and why these formulas work, and apply the formulas to model geometric relationships in the real world.

This assignment allows students to demonstrate their conceptual understanding, procedural fluency, and ability to model and solve real-world problems, which is appropriate for the demands of standards KY.HS.G.27. Although the situation provides the geometric figures to use as models for the water tank, students must interpret cones and cylinders in context and apply what they know about them to calculate the estimated volume. Students must also use what they know about these figures and their relationship to the context to determine which is a more accurate model for the tank. Finally, students must draw their own conclusions and provide their own best estimate for the tank's volume. These actions reflect the modeling process required by the standards.

Please note that inclusion of these sample tasks does not represent that this task is endorsed by or rejected by the Kentucky Department of Education. Inclusion of these tasks was for the sole purpose of allowing participants the opportunity to investigate the content standards within the *Kentucky Academic Standards for Mathematics* more closely. All tasks were selected from https://tntp.org/student-work-library.

Practice Standards

The task provides students the opportunity to reason about the volume of a figure that does not have a formula that they know. This gives students a chance to engage with Mathematical Practice Standard #1 ("Make sense of problems and persevere in solving them") as they determine which information to use—and how to use it—to answer the problem. Students continue to make sense of the problem as they determine how to justify which model may provide the most accurate prediction and to make their own conclusion about the estimated volume. The task also provides opportunities for students to engage with Mathematical Practice Standard #4 ("Model with mathematics") and Mathematical Practice Standard #5 ("Use appropriate tools strategically") as they may use volume formulas, drawings, and other tools to reason and justify their conclusions.

Sample Task 2:

This assignment is partially aligned to the standards.

Overview

Students find the measures of unknown angles and arcs formed by circles, their chords and radii, and tangent segments or rays. Although the assignment aligns with the math content of the standards, it does not engage students in appropriate types of mathematical thinking.

Related Standards

We looked at how well the assignment aligned to the following standard:

KY.HS.G.16: Identify and describe relationships among angles and segments within the context of circles involving:

- KY.HS.G.16.a: Recognize differences between and properties of inscribed, central and circumscribed angles.
- KY.HS.G.16.b: Understand the relationships between inscribed angles and the diameter of a circle.
- KY.HS.G.16.c: Understand the relationship between the radius of a circle and the line drawn through the point of tangency on that radius.

Why is this assignment partially aligned?

The assignment does address the mathematical concepts of standard KY.HS.G.16, as it deals with the relationships among inscribed (and circumscribed) angles, radii, and chords. Students identify the appropriate relationship(s) in order to set up equations and solve for unknown values.

Although the mathematical content of the assignment is aligned to standard KY.HS.G.16, the assignment does not require students to engage with the content in ways required by the standards. Specifically, students are not asked to demonstrate their conceptual understanding of these relationships: It asks students to solve for unknown values, but it doesn't ask them to explain how they arrived at their answers or describe the relationships between the angles formed by chords, radii, and segments tangent to the circles.

Practice Standards

High school geometry standards often require students to go beyond simple computations. For example, standard KY.HS.G.16 requires students to identify and describe the relationships among angles and segments in circles, which allows them to engage in Mathematical Practice Standard #3 ("Construct viable arguments and critique the reasoning of others"). A written explanation of the relationships between the angles formed by chords, radii, and segments

tangent to the circles would allow students to exhibit their understanding of the relationships, as well as their ability to use accurate terminology, symbols, and definitions, as required by Mathematical Practice Standard #6 ("Attend to precision").

Sample Task 3:

This assignment is weakly aligned to the standards.

Overview

Students find unknown values of angles using relationships between the angles of a triangle, vertical angles, and linear pairs of angles. The assignment is weakly aligned with high school geometry standards because it involves applying, rather than proving, theorems and is more closely aligned with seventh- and eighth-grade standards.

Related Standards

We looked at how well the assignment aligned to the following standards:
KY.HS.G.6: Apply theorems for lines, angles, triangles, parallelograms.
KY.HS.G.7: Prove theorems about geometric figures.
KY.HS.G.7.a: Construct formal proofs to justify theorems for lines, angles and triangles.

Why is this assignment weakly aligned?

The Kentucky Academic Standard's Clarifications for KY.HS.G.6 state that "students use previously learned definitions, theorems, postulates and properties of lines, angles, triangles and parallelograms to draw conclusions and to make inferences." This assignment requires students to repeatedly apply a principle that should have already been established in eighth grade (standard KY.8.G.5): that the interior angles of a triangle add up to 180°. Some problems also ask students to apply concepts about supplementary, complementary, vertical, and adjacent angles, which is more appropriate for seventh grade (standard KY.7.G.5).

A high school-level assignment aligned to this content would require students to prove the relationships of angle measures in triangles by writing verbal explanations and naming the mathematical properties that are the basis for solving problems of these types.
High school geometry standards ask students to reason formally about geometric relationships and to apply them in modeling contexts, where geometric principles are applied to authentic real-world scenarios. Students at this grade level should be able to construct careful, mathematically sound proofs and have the chance to connect their mathematical reasoning to authentic contexts. Although it is not unreasonable to access previously learned content in high school assignments, when doing so, students should be asked to reason with the content at a higher level than was required in middle school.

Practice Standards

High school geometry standards about proving theorems lend themselves to Mathematical Practice Standard #3 ("Construct viable arguments and critique the reasoning of others"). The assignment, however, does not ask students to explain their reasoning, nor does it connect to any real-world scenario; instead, it asks students to write and solve equations that are more suited for middle school.

*Please note that inclusion of these sample tasks does not represent that this task is endorsed by or rejected by the Kentucky Department of Education. Inclusion of these tasks was for the sole purpose of allowing participants the opportunity to investigate the content standards within the *Kentucky Academic Standards for Mathematics* more closely. All tasks were selected from https://tntp.org/student-work-library.

Please note that inclusion of these sample tasks does not represent that this task is endorsed by or rejected by the Kentucky Department of Education. Inclusion of these tasks was for the sole purpose of allowing participants the opportunity to investigate the content standards within the *Kentucky Academic Standards for Mathematics* more closely. All tasks were selected from https://tntp.org/student-work-library.